Lemma:- Suppose ΔDEF is the orthic triangle of acute ΔABC with orthocentre H. Then,

(i) Points A, E, F, H lie on a circle with diameter $\overline{AH}$

(ii) Points B, F, E, C lie on a circle with diameter $\overline{BC}$

(iii) H is the incentre of ΔDEF

Proof:- (iii) $\angle BEF = x$

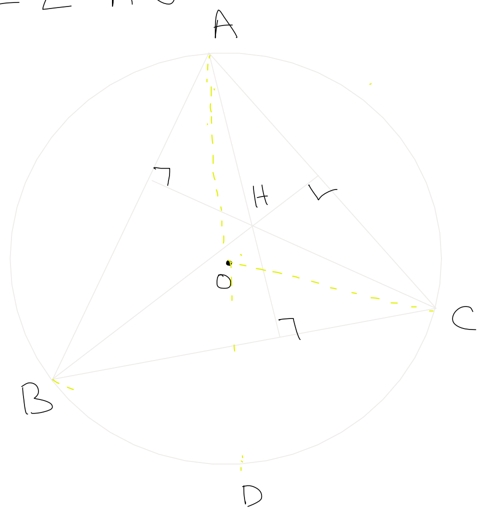
$\angle FAH = \angle FEH = x$

$\angle FAD = \angle FCD = x$

In DHEC, $\angle DEH = \angle HCD = x$ $\Rightarrow \angle FEH = \angle HED$

Similarly for other angles.

Q> Let O and H denote the circumcentre and orthocentre of an acute ΔABC, respectively. Show that $\angle BAH = \angle CAO$



O is circumcentre i.e, centre of the circle.

Ans:- We need to show $\angle CAO = \angle BAH \Rightarrow \angle BAO = \angle CAH$

$\angle BAH = 90^\circ - \angle ABC$

Ans.

$\angle BAH = 90° - \angle ABC$

$\angle CAO = \angle OCA$

$\angle AOC = 2(\angle ABC)$

$$\angle CAO = \frac{(180° - \angle AOC)}{2} = \frac{180° - 2(\angle ABC)}{2}$$

$$= 90° - \angle ABC = \angle BAH$$

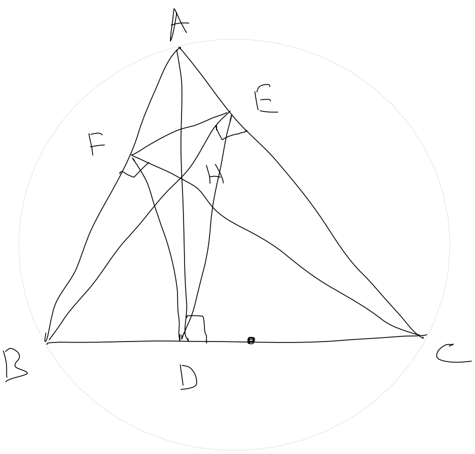
HomeWork

Q) Show that

$\triangle AEF$, $\triangle BFD$, $\triangle CDE$

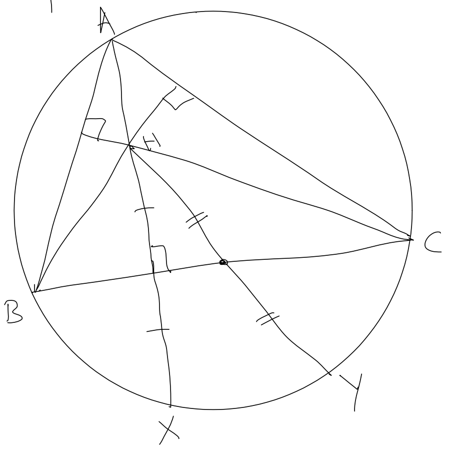
are similar to

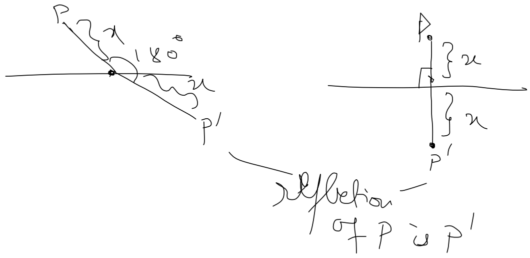
$\triangle ABC$.



HomeWork

Lemma - Let H be the orthocentre of $\triangle ABC$. Let X be the reflection of H over $\overline{BC}$ and Y be the reflection of H over midpoint of $\overline{BC}$





(a) Show that X lies on circumcircle of $\triangle ABC$ (also written as (ABC))

(b) Show that AY is a diameter of (ABC)